B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012
Fourth Semester
Mechanical Engineering
MA 2266/181402/MA 42/MA 1254/10177 SN 401/080120014 — STATISTICS AND NUMERICAL METHODS
(Common to Automobile Engineering and Production Engineering)
(Regulation 2008)
(Common to PTMA 2266 – Statistics and Numerical methods for B.E. (Part-Time)
Second Semester – Production Engineering – Regulation 2009)
Time: Three hours Maximum: 100 marks

Statistical tables may be permitted.
Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are null and Alternate hypothesis?

2. Give the formula for the $\chi^2$-test of independence for $\begin{array}{cc} a & b \\ c & d \end{array}$.

3. State the basic principles of design of Experiments.

4. Define : RBD.

5. Mention the order and condition for the convergence of Newton-Raphson method.

6. Compare Gauss elimination and Gauss-Jacobi methods.

7. What is the need of Newton’s and Lagrange’s interpolation formulae?

8. Find the area under the curve passing through the points (0, 0), (1, 2), (2, 2.5),
(3, 2.3), (4, 2), (5, 1.7) and (6, 1.5)
9. Bring out the merits and demerits of Taylor series method.

10. Find $Y(0.1)$ by Euler's method, if \( \frac{dy}{dx} = x^2 + y^2 \), $y(0) = 1$.

**PART B — (5 x 16 = 80 marks)**

11. (a) (i) A machine puts out 16 imperfect articles in a sample of 500. After it was overhauled, it puts out 3 imperfect articles in a sample of 100. Has the machine improved in its performance?

(ii) Test whether there is any significant difference between the variances of the populations from which the following samples are taken:

Sample I: 20 16 26 27 23 22
Sample II: 27 33 42 35 32 34 38

Or

(b) (i) A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls.

(ii) A sample of 10 boys had the I.Q.'s: 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q. may be 100.

12. (a) The sales of 4 salesmen in 3 seasons are tabulated here. Carry out an analysis of variance.

<table>
<thead>
<tr>
<th>Salesmen</th>
<th>Seasons</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td></td>
<td>36</td>
<td>36</td>
<td>21</td>
<td>35</td>
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<tr>
<td>Winter</td>
<td></td>
<td>28</td>
<td>29</td>
<td>31</td>
<td>32</td>
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<tr>
<td>Monsoon</td>
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<td>26</td>
<td>28</td>
<td>29</td>
<td>29</td>
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</tbody>
</table>

Or

(b) A farmer wishes to test the effect of 4 fertilizers A, B, C, D on the yield of wheat. The fertilizers are used in a LSD and the result are tabulated here. Perform an analysis of variance.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fertilizer</td>
<td>18</td>
<td>21</td>
<td>25</td>
<td>11</td>
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<tr>
<td></td>
<td>22</td>
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<td>10</td>
<td>17</td>
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13. (a) (i) Solve the following equations by Gauss elimination method:
\[ x + y + z = 9, \quad 2x - 3y + 4z = 13, \quad 3x + 4y + 5z = 40. \]

(ii) Find the dominant eigen value of \[
\begin{pmatrix}
1 & 3 & -1 \\
3 & 2 & 4 \\
-1 & 4 & 10
\end{pmatrix}
\]
by power method.

Or

(b) (i) If \[ A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix} \], find \( A^{-1} \) by Gauss-Jordan method.

(ii) Solve the following equations by Gauss-Seidal method:
\[ x + y + 54z = 110, \quad 27x + 6y - z = 85, \quad 6x + 15y + 2z = 72. \]

14. (a) (i) Find \( y(22) \), given that
\[
\begin{align*}
x &: 20 \quad 25 \quad 30 \quad 35 \quad 40 \quad 45 \\
y(x) &: 354 \quad 332 \quad 291 \quad 260 \quad 231 \quad 204
\end{align*}
\]

(ii) Evaluate \( \int_{0}^{x} \sin x \, dx \), by trapezoidal and Simpson's \( \left( \frac{1}{3} \right) \) rules by dividing the range into 10 equal parts.

Or

(b) (i) Find the cubic polynomial \( y(x) \) for
\[
\begin{align*}
x &: -1 \quad 0 \quad 2 \quad 3 \\
y(x) &: -8 \quad 3 \quad 1 \quad 12
\end{align*}
\]

(ii) Find \( y'(1) \), if
\[
\begin{align*}
x &: 0 \quad 2 \quad 3 \quad 4 \quad 7 \quad 9 \\
y(x) &: 4 \quad 26 \quad 58 \quad 112 \quad 466 \quad 922
\end{align*}
\]

15. (a) (i) By Taylor series method find \( y(0.1), \ y(0.2) \) and \( y(0.3) \) if
\[
\frac{dy}{dx} = x - y^2, \quad y(0) = 1.
\]

(ii) By modified Euler's method, find \( y(0.1), \ y(0.2) \) and \( y(0.3) \), if
\[
\frac{dy}{dx} = x + y, \quad y(0) = 1.
\]

Or

(b) If \[
\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, \quad y(0) = 1,
\]
find \( y(0.2), \ y(0.4) \) and \( y(0.6) \) by Runge-Kutta method of fourth order and hence find \( y(0.8) \) by Milne's method.